

# Dark Matter Relic Abundance and Light Sterile Neutrinos

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## Abstract

In this paper, we calculate the relic abundance of the dark matter particles when they can annihilate into sterile neutrinos with the mass  $\lesssim 100$  GeV in a simple model. Unlike the usual standard calculations, the sterile neutrino may fall out of the thermal equilibrium with the thermal bath before the dark matter freezes out. In such case, if the Yukawa coupling between Higgs and sterile neutrino  $y_N$  is small, this process gives rise to a larger  $\Omega_{\text{DM}} h^2$  so we need a larger coupling between dark matter and the sterile neutrino for a correct relic abundance.

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## I. INTRODUCTION

The weakly interacting massive particles (WIMPs) are considered as the candidates of the dark matter (For a review, see Ref. [1]). In this scenario, the dark matter particles were produced in the thermal bath of the early universe, and freeze out of the plasma as the temperature drops. It is well-known that the observed dark matter's relic abundance requires its thermally averaged annihilation cross section  $\langle\sigma v\rangle = 2\text{--}3 \times 10^{-26} \text{ cm}^3/\text{s}$  at the freezing-out temperature  $T \sim \frac{m_\chi}{20}$ , which is roughly the typical cross section of the weak interaction. This coincidence is called the "WIMP miracle".

Calculations of the relic abundance of the dark matter involve the Boltzmann equation (For derivation, see Ref. [2]). In the case of the WIMP dark matter, some hypotheses are adopted in order to simplify the equation. One of the most important hypotheses is that the annihilation products of the dark matter fall in thermal equilibrium with the thermal bath rapidly. This is true when the dark matter mainly annihilates into the standard model (SM) particles. However, in many new physics models, the dark matter might mainly annihilate into other beyond-SM particles. In this case, whether this hypothesis is valid needs to be carefully examined.

In the Type I see-saw model [3–7], the right-handed neutrinos ( $N$ ) couple with the left-handed neutrinos  $l^{\pm,0}$  through the Higgs fields  $H$ , and after the Higgs field acquires a vacuum expectation value (VEV), the majorana mass terms of the left-handed neutrino arise through the Type I See-saw Mechanisms. If there is no extra sector, the main processes that can generate the right-handed neutrinos in the early universe are the decay and the inverse decay of the right-handed neutrinos and the Higgs bosons (For an example of calculations, see [8]). As for the simplest Type I See-saw Mechanisms, when the mass of the right-handed neutrino is approximately 100 GeV, the Yukawa coupling constants of the  $N$ - $l^{\pm,0}$ - $H$  couplings  $y_{NlH}$  should be smaller than  $\sim 10^{-6}$  for the correct left-handed neutrino masses, which is too small for the right-handed neutrinos to reach in thermal equilibrium with the thermal bath. Although there are some models[9–15], e.g., inverse see-saw model, or linear see-saw model, that can result in a larger  $y_{NlH} \sim 0.01$ , as the temperature drops, the thermal-averaged production rates of the sterile neutrinos  $\Gamma_P \propto e^{-\frac{m_N}{T}}$  drops rapidly and then the sterile neutrinos decay out of equilibrium.

In the literature, there are some models that the dark matter can annihilate into light

sterile neutrinos [16–24]. However, as we have mentioned before, the sterile neutrinos might not be in thermal equilibrium with the SM particles in the early universe. Thus, the traditional calculations of the relic abundance might be unreliable and the standard Boltzman equation(s) should be modified. In this paper, in order to calculate these non-thermal effects, we rely on a simple model based on Ref. [21]. We focus on the case that the masses of the dark matter and the right-handed neutrinos are less than  $\sim 100$  GeV and the dark matter particles only annihilate to right-handed neutrinos. In this case, the sterile neutrino mainly decays through the three-body final state channels, and as we have calculated in Ref. [25], this scenario can perfectly explain the gamma-ray excess from near the galactic center. We will also show that the nonthermal effects of the right-handed neutrinos can significantly modify the relic abundance when the Yukawa coupling constant  $y_{NIH}$  becomes quite small.

## II. MODEL DESCRIPTION

The model discussed in this paper contains a majorana fermion  $\chi$  and a real-scalar boson  $\phi$ . Both these two fields are SM-singlets and are odd under a dark  $Z_2$  discrete symmetry. The sterile neutrino together with the SM-fields are all even under the  $Z_2$  symmetry. In this paper, we discuss the two cases. In one case there is one majorana right-handed neutrino  $N$ , and in the other case there are a pair of pseudo dirac sterile neutrino Weyl-fields  $N_{1,2}$ .

In the majorana right-handed neutrino case, the general Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}\bar{\chi}(i\gamma^\mu\partial_\mu - m_\chi)\chi + \frac{1}{2}\bar{N}(i\gamma^\mu\partial_\mu - m_N)N + \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi - m_\phi^2\phi^2) \\ & + y_\chi\bar{\chi}N\phi + iy_{\chi 5}\bar{\chi}\gamma^5 N\phi + \frac{\lambda_\phi}{4!}\phi^4 + \lambda_{\phi H}\phi^2 H^\dagger H + (y_{Ni}\phi\bar{N}P_L l_i \cdot H + \text{h.c.}) \\ & + \mathcal{L}_{\text{SM}}, \end{aligned} \tag{1}$$

where  $N^C = N$ ,  $\chi^C = \chi$  are written in the Dirac four-spinor forms,  $l_i$ ,  $i = 1, 2, 3$  are the left-handed lepton doublets of the three generation,  $m_{\chi, \phi, N}$  are the mass terms of the  $\chi$ ,  $\phi$ ,  $N$ ;  $y_{\chi, \chi 5, Ni}$ ,  $\lambda_{\phi, \phi H}$  are the coupling constants, and  $l_i = \begin{bmatrix} \nu_i \\ e_{Li}^- \end{bmatrix}$ ,  $H = \begin{bmatrix} G^+ \\ v + \frac{h+iG^0}{\sqrt{2}} \end{bmatrix}$  are the left-handed lepton doublet and the Higgs field respectively.  $G^+$ ,  $G^0$  are the goldstone bosons which are eaten by the gauge bosons.  $v = 174$  GeV, and  $h$  is the Higgs boson with its mass set to be 125 GeV.  $A \cdot B$  indicates the contraction of two  $SU(2)_L$  doublets, i.e.,  $A \cdot B = A_i(i\sigma_{ij}^2)B_j$ , where  $\sigma^2$  is the second Pauli-matrix.

In Eqn. (1), all the  $y_{\chi, \chi^5, Ni}$ ,  $\lambda_{\phi, \phi H}$  and  $m_{\chi, \phi, N}$  are real numbers. In fact,  $y_\chi$  and  $y_{\chi^5}$  are respectively the real part and the imaginary part of a single coupling constant  $(y_\chi + iy_{\chi^5})\chi_w \cdot N_w \phi + \text{h.c.}$ , where  $\chi_w$  and  $N_w$  are the Weyl components of the  $\chi$  and  $N$  fields. We should note that all the complex phases in the  $m_{\chi, N}$  and  $y_{Ni}$  can be rotated away by redefining the fields and the  $y_\chi + iy_{\chi^5}$ . However, for simplicity, in the numerical calculations we just omit the  $y_{\chi^5}$  and set it to be zero.

In the pseudo-Dirac sterile neutrino case, the general Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}\bar{\chi}(i\gamma^\mu\partial_\mu - m_\chi)\chi + \overline{N_D}(i\gamma^\mu\partial_\mu - m_{N_D})N_D + \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi - m_\phi^2\phi^2) \\ & + (\mu_1\overline{N_D^C}P_LN_D + \mu_2\overline{N_D^C}P_RN_D + \text{h.c.}) + \frac{\lambda_\phi}{4!}\phi^4 + \lambda_{\phi H}\phi^2H^\dagger H \\ & + (y_{\chi D}\bar{\chi}N_D\phi + iy_{\chi D^5}\bar{\chi}\gamma^5N_D\phi + y_{Ni}\phi\overline{N}P_Ll_i \cdot H + y_{NCi}\phi\overline{N^C}P_Ll_i \cdot H \\ & + \text{h.c.}) + \mathcal{L}_{\text{SM}}, \end{aligned} \quad (2)$$

where  $N_D = \begin{bmatrix} N_1 \\ i\sigma^2 N_2^* \end{bmatrix}$  is a Dirac four-spinor composed of two Majorana fields  $N_1$  and  $N_2$ ,  $m_D$  is the Dirac mass term of the sterile neutrinos, and  $\mu_{1,2}$  are the Majorana mass terms,  $y_{\chi D}$ ,  $y_{\chi D^5}$ ,  $y_{Ni}$ ,  $y_{NCi}$  are the Yukawa coupling constants which is different from the Eqn. (1). This time, we can redefine fields in order for the  $m_\chi$ ,  $m_{N_D}$  and  $y_{Ni}$  to be real numbers, while the  $y_{\chi D}$ ,  $y_{\chi D^5}$ ,  $y_{NCi}$ ,  $\mu_1$ ,  $\mu_2$  might be complex numbers and their phases cannot be rotated away generally. In this paper, for simplicity, we only discuss the case that all these numbers are real and  $y_{\chi D^5} = 0$ .

Generally speaking,  $\mu_{1,2}$  and  $y_{NCi}$  terms violate the lepton number and cause the mass splitting of the two components of the  $N_D$ . If  $y_{NCi} = 0$  and  $\mu_{1,2} \ll m_{N_D}$ , this is called the “inverse see-saw”, and if  $\mu_{1,2} = 0$  and  $y_{NCi} \neq 0$ , this is called the “linear see-saw”. Thus, the masses of the left-handed neutrino are mainly decided by the strength of the lepton number violating terms and  $y_{Ni}$  can be much larger than the simplest see-saw models. Generally speaking, the smallness of the left-handed neutrino masses require the smallness of the lepton number violating terms  $\mu_{1,2}$  and  $y_{NCi}$ . Therefore, in the early universe, the effects of the  $\mu_{1,2}$  and  $y_{NCi}$  terms are negligible, so these parameters are set zero and  $N_D$  is regarded as a Dirac fermion during the calculation processes.

### III. CALCULATION OF THE DARK MATTER'S RELIC ABUNDANCE

The calculations of the relic abundance of the dark matter are based on the Boltzmann equations (We have derived the equations in this paper by following the hints described in Ref. [8, 26]). A full solution to the Boltzmann equations should involve the evolutions to the distribution functions of the particles in principle. However, usually we assume that the elastic scatterings are fast enough as the particles in the thermal bath can maintain kinetic equilibrium. For simplicity, we only consider the case that  $m_\phi > m_\chi + m_{N(D)}$ . In the Majorana right-handed neutrino case, the Boltzmann equations are given by

$$\begin{aligned}
sH z \frac{dY_\chi}{dz} &= -\langle\sigma v\rangle_{\chi\chi\rightarrow NN} Y_{\chi eq}^2 s^2 \left( \frac{Y_\chi^2}{Y_{\chi eq}^2} - \frac{Y_N^2}{Y_{N eq}^2} \right) - \langle\sigma v\rangle_{\chi\chi\rightarrow\phi\phi} Y_{\chi eq}^2 s^2 \left( \frac{Y_\chi^2}{Y_{\chi eq}^2} - \frac{Y_\phi^2}{Y_{\phi eq}^2} \right) \\
&\quad - \langle\sigma v\rangle_{\chi\phi\rightarrow\text{allSM}} s^2 (Y_\chi Y_\phi - Y_{\chi eq} Y_{\phi eq}) - \bar{\Gamma}_{\phi\rightarrow\chi N} Y_{\phi eq} s \left( \frac{Y_\chi Y_N}{Y_{\chi eq} Y_{N eq}} - \frac{Y_\phi}{Y_{\phi eq}} \right), \\
sH z \frac{dY_\phi}{dz} &= -\langle\sigma v\rangle_{\phi\phi\rightarrow NN} Y_{\phi eq}^2 s^2 \left( \frac{Y_\phi^2}{Y_{\phi eq}^2} - \frac{Y_N^2}{Y_{N eq}^2} \right) - \langle\sigma v\rangle_{\phi\phi\rightarrow\chi\chi} Y_{\phi eq}^2 s^2 \left( \frac{Y_\phi^2}{Y_{\phi eq}^2} - \frac{Y_\chi^2}{Y_{\chi eq}^2} \right) \\
&\quad - \langle\sigma v\rangle_{\phi\phi\rightarrow\text{allSM}} s^2 (Y_\phi^2 - Y_{\phi eq}^2) - \langle\sigma v\rangle_{\chi\phi\rightarrow\text{allSM}} s^2 (Y_\chi Y_\phi - Y_{\chi eq} Y_{\phi eq}) \\
&\quad - \bar{\Gamma}_{\phi\rightarrow\chi N} Y_{\phi eq} s \left( \frac{Y_\phi}{Y_{\phi eq}} - \frac{Y_\chi Y_N}{Y_{\chi eq} Y_{N eq}} \right), \\
sH z \frac{dY_N}{dz} &= -\langle\sigma v\rangle_{NN\rightarrow\chi\chi} Y_{N eq}^2 s^2 \left( \frac{Y_N^2}{Y_{N eq}^2} - \frac{Y_\chi^2}{Y_{\chi eq}^2} \right) - 2\langle\sigma v\rangle_{NN\rightarrow\phi\phi} Y_{N eq}^2 s^2 \left( \frac{Y_N^2}{Y_{N eq}^2} - \frac{Y_\phi^2}{Y_{\phi eq}^2} \right) \\
&\quad - \bar{\Gamma}_N s (Y_N - Y_{N eq}),
\end{aligned} \tag{3}$$

where the  $Y_A = \frac{n_A}{s}$  is the actual number of the constituent  $A$  per-comoving-volume, and the  $Y_{A eq} = \frac{n_{A eq}}{s}$  is the equilibrium number of the constituent  $A$  per-comoving-volume,  $n_{A(eq)}$  is the (equilibrium) number density of the constituent  $A$ ,  $s$  is the entropy density,  $z = \frac{m_\chi}{T}$ , and  $T$  is the temperature,  $H$  is the Hubble constant.  $\langle\sigma v\rangle_{AB\rightarrow CD}$  is the thermally averaged cross section times velocity

$$\langle\sigma v\rangle_{AB\rightarrow CD} = \frac{1}{(1 + \delta_{CD}) n_A n_B} \frac{g_A g_B T}{32\pi^4} \int ds' s'^{\frac{3}{2}} K_1 \left( \frac{\sqrt{s'}}{T} \right) \lambda \left( 1, \frac{m_A^2}{s'}, \frac{m_B^2}{s'} \right) \sigma_{AB\rightarrow CD}(s'), \tag{4}$$

where  $\delta_{CD} = 1(0)$  if  $C$  and  $D$  are identical(different) particles,  $g_A$  and  $g_B$  are the degrees of freedoms of particle  $A$  and  $B$ ,  $K_1$  is a Bessel function,  $\sigma_{AB\rightarrow CD}(s')$  is the cross section of the process  $AB \rightarrow CD$  if the total energy in the center of mass frame is  $\sqrt{s'}$ .

The definition of the  $\bar{\Gamma}_{\phi \rightarrow \chi N}$  is given by

$$\bar{\Gamma}_{\phi \rightarrow \chi N} = \frac{K_1(\frac{m_\phi^t}{T})}{K_2(\frac{m_\phi^t}{T})} \Gamma_{\phi \rightarrow \chi N}, \quad (5)$$

where  $m_\phi^t$  is the thermal mass of  $\phi$  which will be defined later. The  $\bar{\Gamma}_N$  is a little bit complicated. We need to consider the decay/inverse-decay processes  $N \rightarrow h^{\pm 0} l$  or  $h^{\pm 0} \rightarrow l N$ . However, as the temperature drops below the electroweak symmetry breaking (EWSB) critical temperature  $T_c$ , we should consider the processes  $N \leftrightarrow W^\pm/Z/h$ . In this paper, we adopt the approximation method described in Ref. [27] to calculate the  $N \rightarrow h^{\pm 0} l$  or  $h^{\pm 0} \rightarrow l N$  with all the four states of the Higgs doublets having the mass  $m_h(T)$  of the Higgs boson below  $T_c$ . If  $m_N > m_h(T)$ ,

$$\tilde{\Gamma}_N = \frac{K_1(\frac{m_N}{T})}{K_2(\frac{m_N}{T})} \Gamma_{N \rightarrow H+l}, \quad (6)$$

while  $m_N < m_h(T)$ ,

$$\tilde{\Gamma}_N = \frac{Y_{Heq}}{Y_{Neq}} \frac{K_1(\frac{m_h(T)}{T})}{K_2(\frac{m_h(T)}{T})} \Gamma_{H \rightarrow Nl}. \quad (7)$$

However, when  $T \ll m_h(0 \text{ GeV}) = 125 \text{ GeV}$ ,  $\tilde{\Gamma}_N$  is severely suppressed by a factor of  $e^{\frac{-2m_h+m_N}{T}}$ . Once it is less than the  $\frac{K_1(\frac{m_N}{T})}{K_2(\frac{m_N}{T})} \Gamma_{N \rightarrow h^*/W^*/Z^*l}$  where  $\Gamma_{N \rightarrow h^*/W^*/Z^*l}$  is calculated at the zero temperature, we set

$$\tilde{\Gamma}_N = \frac{K_1(\frac{m_N}{T})}{K_2(\frac{m_N}{T})} \Gamma_{N \rightarrow h^*/W^*/Z^*l} \quad (8)$$

in order to let the right-handed neutrino decay.

As we have mentioned before, we need to calculate the thermal masses of the particles. The thermal effects on the fermions are small so they are neglected. As for the scalar bosons, the effective potential in a finite temperature  $T$  is calculated to become [28–31]

$$V_{\text{eff}}(h, \phi, T) = \frac{\lambda}{4} h^4 + \frac{1}{2} (\mu^2 + cT^2) h^2 + \frac{\lambda_\phi}{4!} \phi^4 + \frac{\lambda_{\phi h}}{2} \phi^2 h^2 + \frac{1}{2} (m_\phi^2 + c_\phi T^2) \phi^2, \quad (9)$$

where  $\lambda$  is the self-interacting coupling constant of the Higgs boson,  $\mu^2 < 0$  is the mass term at zero temperature of the Higgs potential. The definition of the  $c$  and  $c_\phi$  is given by

$$\begin{aligned} c &= \frac{1}{16} (g_1^2 + 3g_2^2 + 4y_t^2 + 4\frac{m_h^2}{v^2}) + \frac{\lambda_{\phi h}}{12}, \\ c_\phi &= \frac{1}{12} (2y_\chi^2 + 2y_{\chi^5}^2 + \frac{\lambda_\phi}{2} + 4\lambda_{\phi h}). \end{aligned} \quad (10)$$

The critical temperature  $T_c$  of the EWSB is

$$T_c = \sqrt{\frac{-\mu^2}{c}}. \quad (11)$$

Then the temperature dependent masses of the Higgs boson and the scalar  $\phi$  are given by

$$\begin{aligned} m_h(T) &= \begin{cases} \sqrt{\mu^2 + cT^2}, & (T > T_c) \\ \sqrt{-2(\mu^2 + cT^2)}, & (T < T_c) \end{cases}, \\ m_\phi(T) &= \lambda_{\phi h} v(T)^2 + m_\phi^2 + c_\phi T^2, \end{aligned} \quad (12)$$

where

$$v_T = \sqrt{-\frac{\mu^2 + cT^2}{\lambda}}. \quad (13)$$

As for the pseudo-Dirac sterile neutrino case, all the  $N$ 's in the above formulas should be replaced with  $N_D$  and  $\bar{N}_D$ . Note that  $Y_{\bar{N}_D} = Y_{N_D}$  always holds and a summation over the particle and anti-particle should be considered. That is to say, in (3),  $\langle \sigma v \rangle_{AA \rightarrow NN}$  should be replaced with  $\langle \sigma v \rangle_{AA \rightarrow N_D N_D} + \langle \sigma v \rangle_{AA \rightarrow \bar{N}_D N_D} + \langle \sigma v \rangle_{AA \rightarrow \bar{N}_D \bar{N}_D}$ ,  $\bar{\Gamma}_{\phi \rightarrow \chi N}$  should be replaced with  $\bar{\Gamma}_{\phi \rightarrow \chi N_D} + \bar{\Gamma}_{\phi \rightarrow \chi \bar{N}_D}$ ,  $\langle \sigma v \rangle_{NN \rightarrow AA}$  should be replaced with  $\langle \sigma v \rangle_{N_D N_D \rightarrow AA} + \langle \sigma v \rangle_{N_D \bar{N}_D \rightarrow AA}$ . In (10), the  $2y_\chi^2 + 2y_{\chi 5}^2$  should also be replaced with  $4y_{\chi D}^2 + 4y_{\chi D 5}^2$ .

#### IV. NUMERICAL SOLUTIONS AND RESULTS

Now we are going to solve the differential equations (3). Eqn. (3) are a set of stiff equations and fortunately, micrOMEGAs [32] provides us a ready-made function [33, 34] for computing these equations. We also use the CalcHEP [35] embedded in the micrOMEGAs to calculate the  $\langle \sigma v \rangle(s)$  and the widths of the particles. The model file is implemented and output by FeynRules [36]. We adopt the  $g_*$  and  $g_{*S}$  implemented in the micrOMEGAs in order to calculate the Hubble constant  $H = 1.66\sqrt{g_*} \frac{T^2}{M_{pl}}$ , and  $s = \frac{2\pi^2}{45} g_{*S} T^3$ . Here  $M_{pl} = 1.22 \times 10^{19}$  GeV is the Planck energy.

If the  $\lambda_{\phi h}$  is not tuned to be too small, which is the usual case, and when  $T \gg m_\phi$ ,  $\phi$  becomes in thermal equilibrium with the SM particles through the  $\phi\phi \leftrightarrow HH$  interactions, and  $\chi$ ,  $N_{(D)}$  then fall into thermal equilibrium through the  $\phi$ -portal. However, once the temperature drops below the mass of the  $\phi$ , the number density of  $\phi$  rapidly becomes so small that the  $\chi$  and  $N_{(D)}$  decouple from the thermal bath altogether. Finally,  $\chi$ ,  $N_{(D)}$  decouple with each other.

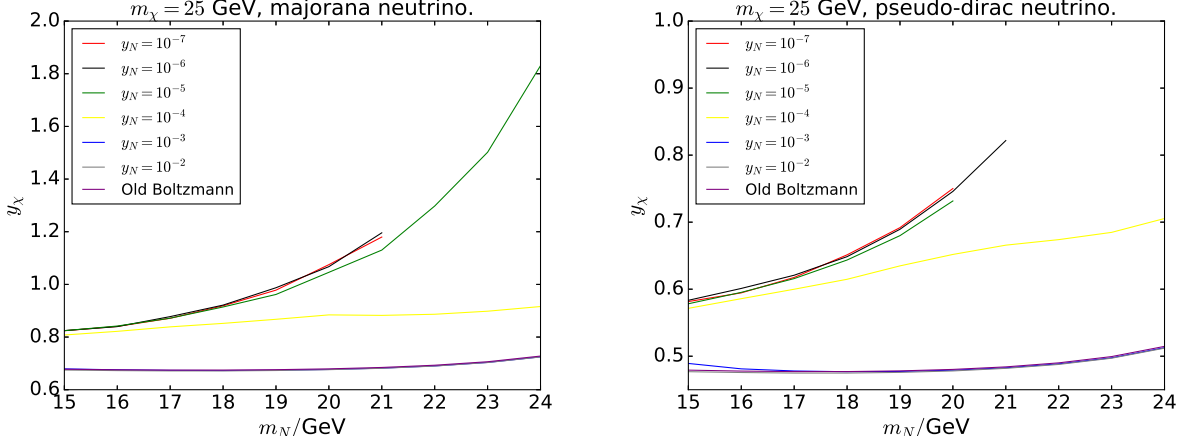


FIG. 1:  $m_\chi = 25$  GeV for the majorana sterile neutrino (left pannel) and pseudo-dirac sterile neutrino (right pannel) case.

During the calculation processes, we simplify the Eqn. (3) by eliminating all the terms involving  $Y_\phi$  once  $Y_\phi < 0.01Y_\chi$ . In order to present our result, we fix  $m_\phi = 180$  GeV,  $\lambda_\phi = 0.5$  and  $\lambda_{\phi H} = 0.45$ ,  $y_{N2} = y_{N3} = 0$ . We plot our results on the  $m_{N_D}$ - $y_\chi$  plane in the different combinations of the values of  $y_N = y_{N1} = 10^{(-7),(-6),(-5),(-4),(-3),(-2)}$  (most of the values are far beyond the current collider bounds, for the relative discussions, see Ref. [37–46]), and  $m_{N(D)} = 25, 52, 76$  GeV. For each  $m_{N(D)}$ , we find one  $y_\chi$  that results in  $0.117 < \Omega_{\text{DM}} h^2 < 0.120$  [47]. We also plot the result calculated by the traditional Boltzmann equation

$$sHz \frac{dY_\chi}{dz} = -\langle \sigma v \rangle_{\chi\chi \rightarrow N(D)N(D)} s^2 (Y_\chi^2 - Y_{\chi eq}^2) \quad (14)$$

for comparison. The results are shown in Fig. 1, 2 and 3. We should note that in these figures, some lines, especially the blue, grey and purple ones may overlap and become indistinguishable. It means that when  $y_N \gtrsim 10^{-3}$ , the standard calculation by the old Boltzmann equation is quite reliable. Note that when  $y_N \lesssim 10^{-5}$  and  $m_{N(D)}$  approaches  $m_\chi$ , the numerical processes of solving the Boltzmann equation becomes so slow that we give up to show the complete results in this case. Therefore, the red and black lines may disappear before they reach the most right-handed side in Fig. 1-3.

In order to have a look at the decoupling processes in detail, we plot the  $z$ -evolution of the  $\frac{Y_\chi}{Y_{\chi eq}}$  and  $\frac{Y_N}{Y_{N eq}}$  in the Fig. 4. From this we can clearly see that before  $z \lesssim 30$ , the  $\chi$  and the sterile neutrino together decouple from the thermal bath, however they are in



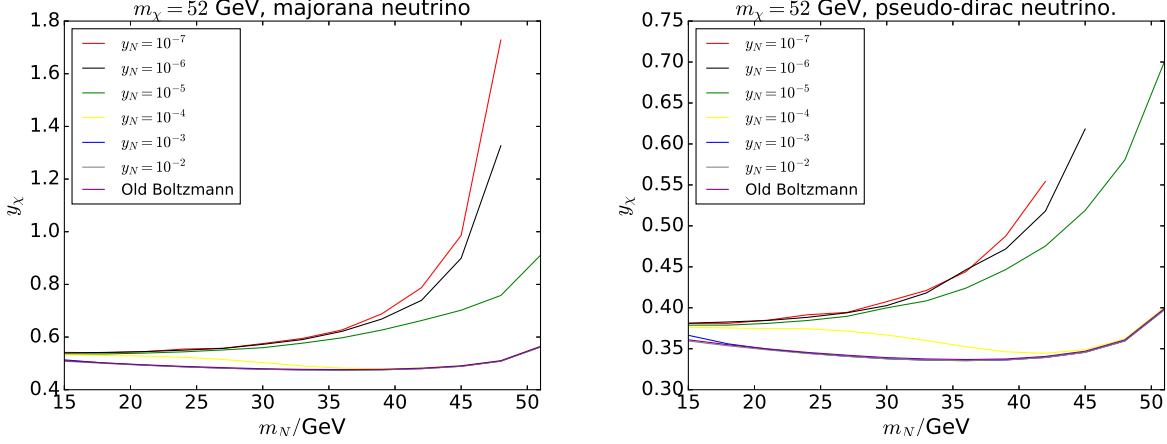


FIG. 2:  $m_\chi = 52$  GeV for the majorana sterile neutrino (left pannel) and pseudo-dirac sterile neutrino (right pannel) case.

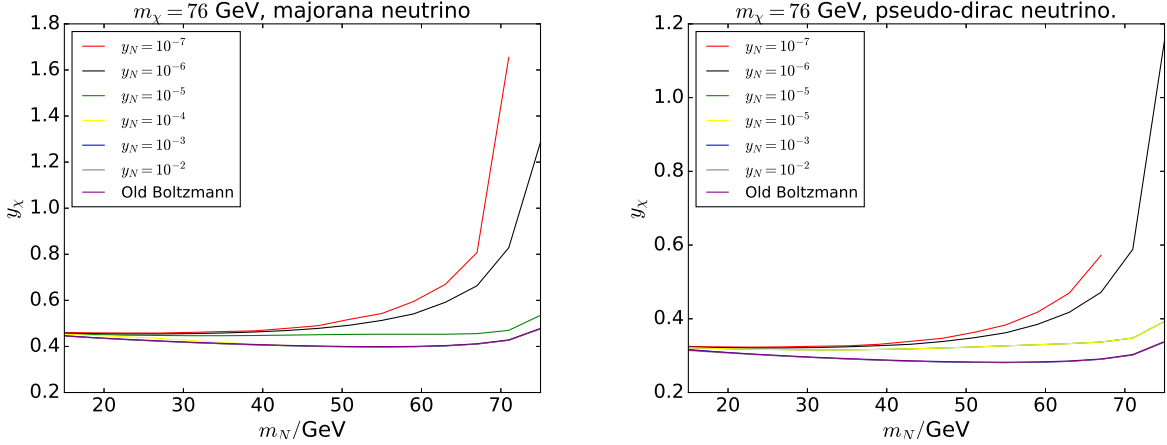


FIG. 3:  $m_\chi = 76$  GeV for the majorana sterile neutrino (left pannel) and pseudo-dirac sterile neutrino (right pannel) case.

thermal equilibrium with each other, so  $\frac{Y_\chi}{Y_{\chi eq}}$  traces the  $\frac{Y_N}{Y_{N eq}}$  very well. Then the  $\chi$  and the  $N$  decouple with each other and  $\chi$  finally freezes out. After that, although  $\frac{Y_N}{Y_{N eq}}$  still arises as the  $z$  accumulates,  $y_N$  actually drops as  $y_{N eq}$  decreases much faster. Usually, if  $y_N$  is small, when  $\chi$  and  $N_{(D)}$  decouples with each other,  $Y_{N_{(D)}}$  is usually larger than  $Y_{N_{(D) eq}}$ , which gives rise to the  $Y_\chi$  at the freeze-out point. That is why we need a larger  $y_\chi$  to suppress the  $Y_{\chi\infty} \propto \Omega_{DM} h^2$  in the case of a small  $y_N \ll 10^{-3}$  as shown in Fig. 1-3.

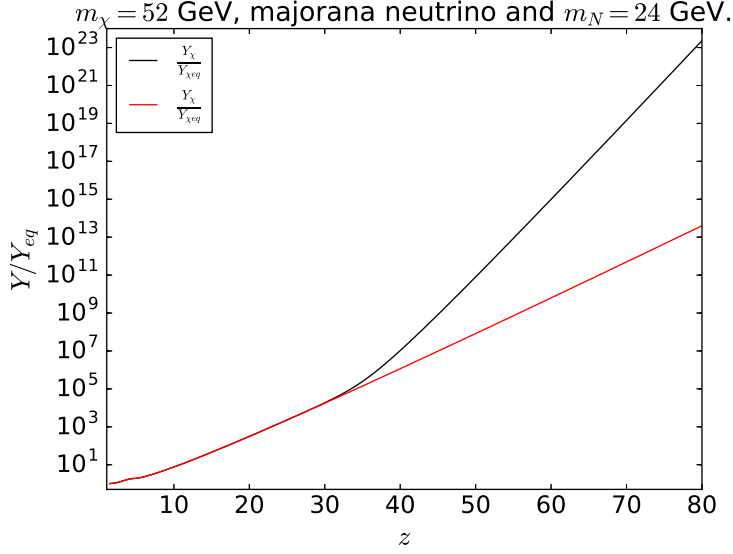


FIG. 4: The  $z$ -evolution of  $\frac{Y_\chi}{Y_{\chi eq}}$  and  $\frac{Y_N}{Y_{N eq}}$  in the benchmark point that  $m_N = 24$  GeV,  $m_\chi = 52$  GeV,  $y_\chi = 0.554$ , and  $y_N = 10^{-7}$ .

## V. DISCUSSIONS

In this paper, we have chosen the fermionic  $\chi$  as the dark matter candidate. In this case, there is no tree-level diagrams contributing to the direct detection processes. However, As has been mentioned in Ref. [20, 21], one-loop diagram will result in  $\bar{\chi}(I, \text{ or } i\gamma^5)\chi H^\dagger H$  operators, which give rise to not only the direct detection processes through exchanging a Higgs boson with the target nucleon, but also lead to the Higgs invisible decays. These effects are all suppressed by the loop factor and are proportional to  $y_\chi^4$ . From the Fig. 1-3, we can see that in order to get an appropriate dark matter relic abundance, we need a larger  $y_\chi^4$  than usual standard calculations. However, there does exist some parameter space that the needed  $y_\chi^4$  is only larger than the usual case of less than one order of magnitude if we adjust  $y_N$ , so we need not to worry whether this scenario was fully ruled out.

Ref. [25] has calculated the galactic center gamma-ray excess in such kind of scenario. In Ref. [25] we have pointed out that an approximately 10-60 GeV sterile neutrino together with a heavier dark matter particle can perfectly explain the the observed spectrum. The annihilation cross section  $\langle\sigma v\rangle$  is within the range  $0.5\text{-}4\times 10^{-26}\text{cm}^3/\text{s}$ . In this paper, we need a larger  $y_\chi^4$ , which means a larger  $\langle\sigma v\rangle$  than the standard WIMP cross section  $\sim 3\times 10^{-26}\text{cm}^3/\text{s}$ . However, as the direct detection case, there are enough room in the parameter space to let

$\langle\sigma v\rangle$  fall in the range of  $0.5\text{--}4 \times 10^{-26}\text{cm}^3/\text{s}$  if we adjust  $y_N$ .

Finally, we should note that although the  $y_N$  might be as small as  $10^{-7}$  in the simplest see-saw model and this will result in a relatively long lifetime. For example, the width  $\Gamma_N \sim 10^{-17}$  GeV when  $m_N \sim 50$  GeV, but this is still much larger compared with the Hubble constant  $H \sim 10^{-22}$  GeV at the BBN temperature  $T \sim 10$  MeV, so the out-of-equilibrium sterile neutrino will decay up before they may have an impact on the big bang nucleon-synthesis (BBN).

## VI. CONCLUSIONS

We have calculated the relic abundance of the dark matter particles when they annihilate into sterile neutrinos with the mass  $m_N < m_\chi \lesssim 100$  GeV. In the model we have relied on, the sterile neutrino will become in thermal equilibrium with the thermal bath when  $T \gg m_{N(D)}$ , however it will decouple with the thermal bath before the dark matter freezes out if  $y_N$  is small. This gives rise to a larger  $\Omega_{\text{DM}}h^2$  so we need a larger coupling between dark matter and the sterile neutrino for a correct relic abundance. In the future, we will continue to dedicate ourselves in some more detailed research in such kind of scenarios.

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